**DISCUSSION PAPER**

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# Gravity Model Estimation: Fixed Effects vs. Random Intercept Poisson Pseudo Maximum Likelihood

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## Abstract

Since the work of Feenstra (2002), the standard Anderson & van Wincoop (2003) Gravity Model has been estimated using a fixed effects approach. However, a fixed effects approach has a major drawback: it does not allow for the estimation of exporter- and importer-invariant variables. Thus, economically relevant variables such as exporter and importer gross domestic product are disregarded. Here, we propose a random intercept model to address this gap. This approach not only provides identical estimates to a fixed effects approach, but also allows for the estimation of exporter- and importer-invariant variables.



JEL: F1, C3

Keywords: Gravity Model Estimation, Poisson Pseudo Maximum Likelihood, Fixed Effects Model, Random Intercept Model.



## Zusammenfassung

Gravity Modell Schätzung: Fixed Effekts vs. Random Intercept Poisson Pseudo Maximum Likelihood

Seit Feenstra (2002) wird das Standard Anderson & van Wincoop (2003) Gravity Modell mittels Fixed-Effects-Ansatzes geschätzt. Der Fixed-Effects-Ansatz hat allerdings den entscheidenden Nachteil, dass er nicht die Schätzung von Exporteur- und Importeur-invarianten Variablen erlaubt. Folglich lassen sich ökonomisch relevante Variablen wie Exporteur- und Importeur-Bruttosozialprodukt nicht berücksichtigen. Wir empfehlen ein Random-Intercept-Modell anstatt. Dieser Ansatz liefert nicht nur die identischen Schätzer wie ein Fixed-Effects-Modell, sondern erlaubt auch die Schätzung von Exporteur- und Importeur-invarianten Variablen.



JEL: F1, C3

Schlüsselwörter: Gravity Modell, Poisson Pseudo Maximum Likelihood, Fixed Effects Modell, Random Intercept Modell.

**Abstract.** Since FEENSTRA (2002), the standard ANDERSON & VAN WINCOOP (2003) Gravity Model is estimated by a fixed effects approach. Still, a fixed effects approach has a major drawback: it does not allow for the estimation of exporter- and importer-invariant variables. Thus, economically relevant variables such as exporter and importer gross domestic product are disregarded. Here, we propose a random intercept model instead. This approach not only gives identical estimates like a fixed effects approach but also allows for the estimation of exporter- and importer-invariant variables.

**Keywords:** Gravity Model Estimation, Poisson Pseudo Maximum Likelihood, Fixed Effects Model, Random Intercept Model

Similar to the ongoing discussion about the appropriate theoretical specification of gravity models (ANDERSON, 1979), debate continues about the consistent estimation of gravity models (SANTOS SILVA & TENREYRO, 2006). The ANDERSON & VAN WINCOOP (2003) (AvW) Model is now state-of-the-art; these authors show that one must account for outward and inward multilateral resistance, otherwise estimation results are biased. Not accounting for outward and inward multilateral resistance leads to an omitted variable bias. The AvW Model is defined as follows:

$$X_{ij} = \left(\frac{Y_i Y_j}{Y^w}\right)\left(\frac{\tau_{ij}}{\Pi_i P_j}\right)^{1-\sigma}$$

with

$$\Pi_i^{1-\sigma} = \sum_j \left(\frac{\tau_{ij}}{P_j}\right)^{1-\sigma} \frac{Y_j}{Y^w}$$

$$P_j^{1-\sigma} = \sum_i \left(\frac{\tau_{ij}}{\Pi_i}\right)^{1-\sigma} \frac{Y_i}{Y^w},$$

where $X_{ij}$ is the export value from exporter i to importer j, $\tau_{ij}$ are bilateral trade costs between i and j, and $\Pi_i$ and $P_j$ are outward and inward multilateral resistance, respectively. The variables $Y_i$, $Y_j$, and $Y^w$ are i's, j's, and world gross domestic product, while σ indicates the elasticity of substitution.

Although ANDERSON & VAN WINCOOP recommend estimating their model using a nonlinear programming approach, it is now standard practice to estimate the AvW Model using a fixed effects approach. In Feenstra's seminal article 'Border Effects and the Gravity Equation: Consistent Methods for Estimation' (SCOTTISH JOURNAL OF POLITICAL ECONOMY, 49, 5: pp. 491-06), he shows that one does not commit an error when estimating the AvW Model using a fixed effects approach. Rather, the coefficients of source and destination fixed effects are consistent measures for outward and inward multilateral resistance.

In practice, the AvW Model is usually estimated by a fixed-effects Poisson Pseudo Maximum Likelihood (PPML) approach (SANTOS SILVA & TENREYRO, 2006). The PPML has several advantages compared to other estimators; it deals appropriately with heteroscedasticity, model misspecification, and excess zeros. The fixed effects approach, however, has one major drawback: it does not allow for the estimation of exporter- and importer-invariant variables, i.e., variables such as exporter and importer gross domestic product (GDP) cannot be estimated when a fixed effects approach is applied. The lack of within-group variation in the former variables makes it impossible to identify them. This is a problem because variables such as GDP and so on are economically interesting in trade analysis. It would therefore be important to understand how they impact trade.

One can argue in favor of a random effects model. However, a random effects model has other shortcomings. Commonly, the independence assumption of residuals and covariates is not fulfilled. An alternative is a random intercept model, which combines the advantages of a fixed effects model and that of a random effects model (RABE-HESKETH & SKRONDAL, 2012). A random intercept model uses both within-group variation and between-group variation for estimating. Hence, a random intercept model not only yields estimates that are identical to a fixed effects model, but also allows for the estimation of exporter- and importer-invariant variables.[1] The latter particularly increases the explanatory power of the standard AvW Model.

Implementing PPML is as simple to implement as a random intercept PPML. The corresponding random intercept PPML model is given as follows:

$$X_{ij} = \exp\left(\beta_0 + \beta_1\left(\tau_{ij} - \bar{\tau}_{i\cdot} - \bar{\tau}_{\cdot j} + \bar{\tau}_{\cdot\cdot}\right) + \beta_2 Y_i + \beta_3 Y_j + \zeta_{0i} + \zeta_{0j}\right) + e_{ij}\,,$$

where $\beta_0$ indicates a constant. The Random exporter intercept is $\zeta_{0i} \sim N(0, \sigma^2_{\zeta_{0i}})$, the random importer intercept is $\zeta_{0j} \sim N(0, \sigma^2_{\zeta_{0j}})$, and the error term is $e_{ij} \sim N(0, \sigma^2_e)$. As with a fixed effects model, some data transformations are required in advance. For the fixed effects portion, variables associated with bilateral trade costs have to be transformed as follows: $\tau_{ij}^{within} = \tau_{ij} - \bar{\tau}_{i\cdot} - \bar{\tau}_{\cdot j} + \bar{\tau}_{\cdot\cdot}$, where bars and dots indicate the corresponding means over importer and/or exporter, respectively.[2] As random intercept models belong to the broader class of mixed effects models, standard estimation routines can be used.[3] In R, the corresponding function is glmer included in the R package lme4 (BOLKER, 2014). The R command is

$$\mathrm{glmer}\left(X_{ij} \sim \tau_{ij}^{within} + Y_i + Y_j + (1|exp_{id}) + (1|imp_{id}), \mathrm{family} = \mathrm{poisson}()\right),$$

where $exp_{id}$ and $imp_{id}$ indicate exporter and importer identification. Estimates can still be interpreted as before, i.e., variables in logs as elasticities and variables in levels as semi-elasticities.

For illustration, we revisit the famous ANDERSON & VAN WINCOOP example where border effects within and between United States and Canada are analyzed. Feenstra uses the same dataset as a benchmark.[4]

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1 If one only focuses on positive trade flows, the approximation approach of trade-cost effects by BAIER & BERGSTRAND (2009) is alternative. This approach, however, is not applicable to zero trade flows.

2 For a panel model, a time-varying random intercept model is required (BALDWIN & TAGLIONI, 2007). The corresponding data transformation is $\tau_{ijt}^{within,time-varying} = \tau_{ijt} - \bar{\tau}_{i\cdot t} - \bar{\tau}_{\cdot jt} + \bar{\tau}_{\cdot\cdot t}$.

3 A mixed effects approach is also recommended by PROENCA et al. (2012). The authors, however, favor a semi-mixed effects approach. Here, the assumption of identical preferences is skipped.

4 For details, see FEENSTRA (2002) and ANDERSON & VAN WINCOOP (2003).

**Table 1: Comparison Gravity Estimators**

| | Without Zeros | | With Zeros | |
|---|---|---|---|---|
| | Fixed Effects Model | Random Intercept Model | Fixed Effects PPML | Random Intercept PPML |
| | (1) | (2) | (3) | (4) |
| Year of data | 1993 | 1993 | 1993 | 1993 |
| Independent variables | | | | |
| $\ln Y_i$ | 1 | 1.13*** (0.05) | 1 | 0.73*** (0.11) |
| $\ln Y_j$ | 1 | 0.97*** (0.03) | 1 | 0.91*** (0.03) |
| $\ln d_{ij}$ | -1.25*** (0.04) | -1.25*** (0.04) | -1.01*** (0.04) | -1.01*** (0.00) |
| Indicator border | -1.55*** (0.07) | -1.55*** (0.06) | -1.24*** (0.10) | -1.24*** (0.00) |
| Observations | 1511 | 1511 | 1560 | 1560 |

Source: Own calculations.

Note: Standard errors appear in parentheses. Fixed effects and random intercepts are not presented.

Table 1 contrasts Feenstra's fixed effects model (column 1) with a random intercept model (column 2), a fixed effects PPML approach (column 3), and a random intercept PPML approach (column 4). As argued above, for the fixed effects portion, one obtains identical estimates for distance and common border for log-linear models (column 1-2) and for multiplicative models (column 3-4). Further, for the random intercept models (column 2 and 4) one also obtains estimates for exporter and importer gross domestic product. Hence, applying random intercept models allows the researcher to gain additional insight into the drivers of bilateral trade.

The analysis, however, is not only restricted to exporter and importer gross domestic product. Rather, other exporter- and importer-invariant variables such as tariffs, infrastructure and so on can also be analyzed. This makes random intercept models a very valuable tool for analyzing policy-relevant variables in a standard AvW Model setting.

## CONCLUSIONS

It is common practice to estimate gravity models using a fixed effects approach. However, such an approach has a major drawback: it does not allow for the estimation of exporter- and importer-invariant variables. The trade impact of economically relevant variables is disregarded. Random intercept models are a solution because they not only use within-group variation but also between group variation for estimation. One therefore obtains estimates for both variant and exporter- and importer-invariant variables.

In future, the standard AvW Model should be estimated by random intercept models. This would significantly increase the scope of questions that can be analyzed in a standard AvW Model setting. Besides bilateral trade costs, the trade impact of policy relevant variables such as tariffs, infrastructure and so can be analyzed. Gravity models thus become more favorable for policy analyses.

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